A short guide to KrakenZ

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Contents

1 Introduction

The purpose of this short guide is to provide some informations on how to use a new root finder for the Kraken normal program. This new root finder is based on winding number integrals and aims at providing a robust solution to the problem of mode detection. This is not a complete version since many changes have been made after this preliminary version but it performs well for configurations of the ocean which are not "critical". It is designed in order to be as fast as possible and can only solve problems with semi-infinite bottom. It means that KrakenZ in its present version is not adequate for the solution of coupled mode problem with a rigid bottom. Another version which will be able to solve all the problems Kraken is able to solve is under construction.

2 The way it is built

In order to get a code which is not too slow, winding number integrals are only implemented for the first mesh. It means that there are used to provide good initial guesses to the secant method which performs very well if it is well feeded. This saves cpu time and no particular difficulties were encountered with this procedure.

It must be noted that winding number integrals require a lot of cputime to be evaluated and that they have some limitations that must be kept in mind. The accuracy of the result obtained depends on the discretization of the contour which is used and on the number of roots which lie inside this contour. Since the roots are evaluated by means of a routine which calculates the roots of a polynomial, it is not possible to get for example 40 roots simultaneously. Therefore the use of a single contour between two branch points wil only be possible for low values of the product of the frequency by the water depth. Several contours will have to be designed. It is then required to control the number of roots which will lie inside each contour.

This evaluation can be done with winding number integrals but it makes the speed of the code to slow down. Therefore the effective depth concept was used to get an approximation of the total number of roots. This is clearly a limitation but it works for most of the configurations that can be encountered. This is the reason why this version is not able to handle couple mode configurations.

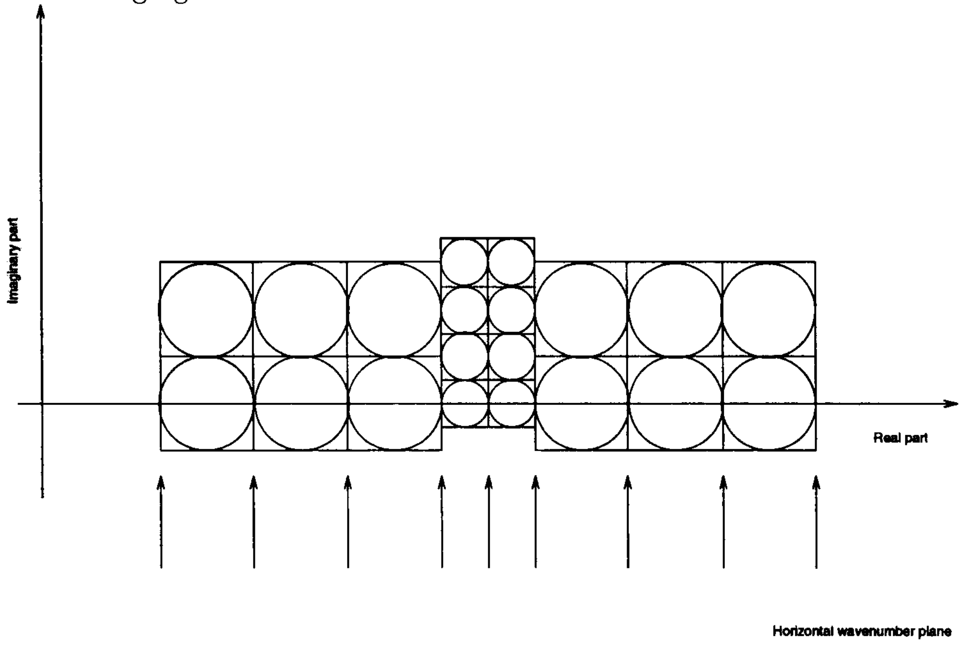
An extension of the effective depth concept was designed in order to take into account modes whose sound speed is less than the minimum sound speed in the water column. See for example the test file named LOVE.ENV.

A special treatment of the EJP cuts was used to avoid crossings of the contours.

The root finder based on winding number integrals performs in the following way :

1. The effective depth concept determines all the areas of the complex plane where a contour will be used
2. For each area, a square which corresponds to this area and a circle which lie inside the square are determined
3. The number of roots which lie inside the square and the circle defined previously is calculated
4. If the number of roots which lie inside the circle is equal to the number of roots which lie inside the square, the circle is used to perform the calculation of the roots or else it is the square which is used

An illustration of the partionning of the complex plane that is performed is given in the following figure.



3 The changes

There is only one change in the environmental file which had to be introduced. It was necessary to consider a minimum range (RMIN) for which the accuracy of the solution is required. This value is closely related to the surface of the complex plane which will be examined. The code will consider modes which have an attenuation of 40 dB at this distance RMIN.

(8) - MAXIMUM/MINIMUM RANGE

Syntax :

RMAX RMIN

In this way, the environmental files designed for KrakenZ can be used with Krakenc.

There is another requirement connected to the use of the effective depth concept. The water medium must correspond to one medium. This means that the first medium must be the entire water column and the sediment begins with the second medium.

4 Additional informations on the new routines

The routine which performs the calculation of winding number integrals is SOLVER.F. In this routine there is three parameters which control the behaviour of the root finder. The first parameter is NCER which is the number of points that is used to discretize a square. Its value is set to 500. The second parameter is NCIR which is the number of points that is used to discretize the circle. The third parameters is MXINTMOD which is the maximum number of modes allowed inside one contour. It controls the partionning of the complex plane which is done i.e. the number of squares which will be used to find all the modes.

5 Test problems

Three other test problems were designed to study shallow water propagation over a complex bottom. They all correspond to shallow water configurations.

The first three examples concern a configuration where there is a lot of modes whose sound speed is less than the sound speed of the water. In order to have a good convergence a fine sampling is required and surprisingly Krakenc have difficulties in converging while KrakenZ has not. Krakenc provides better results with a less fine mesh even if it is not converging. 13 modes are correct with the finest mesh while 21 are correct with a less fine mesh. (See the corresponding Print files). In the second example, the case where the last propagative mode is close to a branch point is analysed. This is a critical configuration for Krakenc. It misses the Scholte mode and the first six propagative modes. It can be noticed that in order to converge the code requires a fine mesh.

The last example corresponds to a less complex configuration. It is a shallow water version of the fluid sediment configuration of Kraken manual with a frequency such that the last propagative mode is close to a branch point. In this configuration the mesh needs not to be as fine as for the previous example. Krakenc still misses modes but performs better.

5.1 Love problem

```
KRAKENC-Love problem

Frequency =   100.0     NMEDIA =   2


    C-LINEAR approximation to SSP
    Attenuation units: dB/mkHz
    VACUUM


     Z           ALPHAR     BETAR       RHO        ALPHAI      BETAI


        ( Number of pts =   1000  RMS roughness =   0.000E+00 )
     0.00        1500.00      0.00      1.00        0.0000     0.0000
   100.00        1500.00      0.00      1.00        0.0000     0.0000

        ( Number of pts =    250  RMS roughness =   0.000E+00 )
   100.00        1650.00    400.00      1.90        0.2000     0.4000
   125.00        1650.00    400.00      1.90        0.2000     0.4000

                                 ( RMS roughness =   0.000E+00 )
    ACOUSTO-ELASTIC half-space
   125.00        1800.00    600.00      2.00        0.5000     1.0000

CLOW =    350.00      CHIGH =    4000.0
RMAX =     1000.00000000000

Number of sources   =            1
  70.00000

Number of receivers =            1
  70.00000

Mesh multiplier   CPU seconds
       1            0.174
       2            0.301
       4            0.140
       8            0.251
      16            0.764
```

```
*** WARNING ***

Error detected in program or subroutine: KRAKENC
Too many meshes needed: check convergence
```

I	K	ALPHA	PHASE SPEED
1	1.564905257	-0.4629535024E-02	401.5057958
2	1.547223356	-0.4703537706E-02	406.0942644
3	1.517560371	-0.4830331953E-02	414.0319838
4	1.475550837	-0.5017560289E-02	425.8196431
5	1.420604378	-0.5280418078E-02	442.2895921
6	1.351988258	-0.5648315959E-02	464.7366772
7	1.269405910	-0.6189706599E-02	494.9705416
8	1.176653840	-0.7157937957E-02	533.9875750
9	1.098223526	-0.8541262865E-02	572.1226290
10	1.023013947	-0.1693924774E-01	614.1837388
11	0.4023335933	-0.1044558037E-03	1561.685480
12	0.3927340794	-0.1465115920E-03	1599.857419
13	0.3627057987	-0.1102372572E-02	1732.309031
14	0.3346967161	-0.1099455865E-01	1877.277250
15	0.3218571469	-0.5337489975E-02	1952.165850
16	0.2432564723	-0.7504884346E-02	2582.946817
17	0.1891431350	-0.1366928457E-01	3321.920886

```
KRAKENZ-Love problem

Frequency =  100.00     NMEDIA =   2


    C-LINEAR approximation to SSP
    Attenuation units: dB/mkHz
    VACUUM


     Z           ALPHAR      BETAR       RHO        ALPHAI      BETAI


          ( Number of pts =  1000  RMS roughness =  0.000E+00 )
     0.00        1500.00      0.00       1.00       0.0000      0.0000
   100.00        1500.00      0.00       1.00       0.0000      0.0000

          ( Number of pts =   250  RMS roughness =  0.000E+00 )
   100.00        1650.00    400.00       1.90       0.2000      0.4000
   125.00        1650.00    400.00       1.90       0.2000      0.4000

                                       ( RMS roughness =  0.000E+00 )
    ACOUSTO-ELASTIC half-space
   125.00        1800.00    600.00       2.00       0.5000      1.0000

CLOW =   350.00      CHIGH =   4000.0
RMIN, RMAX =   0.500000000000000        1000.00000000000

Number of sources    =           1
  70.00000

Number of receivers =           1
  70.00000

Mesh multiplier   CPU seconds
       1            1.38
       2           0.900E-01
       4           0.137

   I          K                ALPHA           PHASE SPEED
   1     1.769048619     -0.5254541008E-02    355.1731276
   2     1.564918878     -0.4629575835E-02    401.5023012
   3     1.547237345     -0.4703582270E-02    406.0905929
   4     1.517575006     -0.4830383694E-02    414.0279908
```

5	1.475566473	-0.5017624596E-02	425.8151308
6	1.420621531	-0.5280505306E-02	442.2842517
7	1.352007838	-0.5648451693E-02	464.7299470
8	1.269430079	-0.6189979409E-02	494.9611175
9	1.176691042	-0.7158749501E-02	533.9706928
10	1.098289062	-0.8541673473E-02	572.0884895
11	1.023047247	-0.1695520089E-01	614.1637471
12	0.8843540613	-0.2972435984E-01	710.4830047
13	0.7101347038	-0.5852239795E-01	884.7878119
14	0.5186426805	-0.1345577218	1211.467074
15	0.4178677301	-0.1000692274E-04	1503.630181
16	0.4148148900	-0.3621321931E-04	1514.696183
17	0.4096651752	-0.6993185409E-04	1533.736741
18	0.4023339943	-0.1044807112E-03	1561.683923
19	0.3927347813	-0.1465731845E-03	1599.854560
20	0.3809444112	-0.2769357161E-03	1649.370649
21	0.3696161565	-0.1521413798E-02	1699.921715
22	0.3627194018	-0.1101521405E-02	1732.244064
23	0.3474937937	-0.1409253781E-02	1808.143173
24	0.3343206274	-0.1136562491E-01	1879.389063
25	0.3214665499	-0.5127952232E-02	1954.537823
26	0.2935955734	-0.4515894038E-02	2140.081757
27	0.2592216614	-0.6420814404E-02	2423.865843
28	0.2191096548	-0.5953527667E-01	2867.598561
29	0.2123856585	-0.1085559467E-01	2958.384926

5.2 Love problem (critical frequency)

```
KRAKENC-Love problem ( Critical frequency )

Frequency =   101.2     NMEDIA =   2


    C-LINEAR approximation to SSP
    Attenuation units: dB/mkHz
    VACUUM


     Z           ALPHAR      BETAR       RHO        ALPHAI      BETAI


         ( Number of pts =   1000  RMS roughness =   0.000E+00 )
     0.00        1500.00      0.00       1.00        0.0000     0.0000
   100.00        1500.00      0.00       1.00        0.0000     0.0000

         ( Number of pts =    250  RMS roughness =   0.000E+00 )
   100.00        1650.00    400.00       1.90        0.2000     0.4000
   125.00        1650.00    400.00       1.90        0.2000     0.4000

                                     ( RMS roughness =   0.000E+00 )
    ACOUSTO-ELASTIC half-space
   125.00        1800.00    600.00       2.00        0.5000     1.0000

 CLOW =    350.00       CHIGH =    4000.0
 RMAX =     1000.00000000000

 Number of sources   =             1
   70.00000

 Number of receivers =             1
   70.00000

 Mesh multiplier   CPU seconds
       1             0.179
       2             0.279
       4             0.150
       8             0.306
      16             0.404
```

```
*** WARNING ***

Error detected in program or subroutine: KRAKENC
Too many meshes needed: check convergence
```

I	K	ALPHA	PHASE SPEED
1	1.583520007	-0.4683456211E-02	401.4680501
2	1.566077631	-0.4756174957E-02	405.9394483
3	1.536819001	-0.4880687103E-02	413.6679002
4	1.495390467	-0.5064261684E-02	425.1282212
5	1.441219712	-0.5321260062E-02	441.1074065
6	1.373582219	-0.5679137450E-02	462.8282754
7	1.292086288	-0.6199727129E-02	492.0203051
8	1.199643814	-0.7101278178E-02	529.9345374
9	1.118026765	-0.8566858089E-02	568.6202777
10	1.048086398	-0.1436380697E-01	606.5651560
11	0.3746231504	-0.1444381304E-02	1696.992534
12	0.3536429673	-0.1168430164E-02	1797.668123
13	0.2883611462	-0.3910729934E-02	2204.640596
14	0.2520849047	-0.7525253326E-02	2521.899081
15	0.2011588738	-0.1286357299E-01	3160.351207

```
KRAKENZ-Love problem ( Critical frequency )

Frequency =  101.18     NMEDIA =   2


    C-LINEAR approximation to SSP
    Attenuation units: dB/mkHz
    VACUUM


     Z          ALPHAR      BETAR       RHO        ALPHAI      BETAI


          ( Number of pts =   1000  RMS roughness =  0.000E+00 )
     0.00        1500.00      0.00       1.00       0.0000      0.0000
   100.00        1500.00      0.00       1.00       0.0000      0.0000

          ( Number of pts =    250  RMS roughness =  0.000E+00 )
   100.00        1650.00    400.00       1.90       0.2000      0.4000
   125.00        1650.00    400.00       1.90       0.2000      0.4000

                                   ( RMS roughness =  0.000E+00 )
    ACOUSTO-ELASTIC half-space
   125.00        1800.00    600.00       2.00       0.5000      1.0000

CLOW =   350.00      CHIGH =   4000.0
RMIN, RMAX =   0.500000000000000        1000.00000000000

Number of sources   =            1
  70.00000

Number of receivers =            1
  70.00000

Mesh multiplier   CPU seconds
       1             1.44
       2            0.103
       4            0.153

    I          K                ALPHA             PHASE SPEED
    1     1.789923391     -0.5316544601E-02     355.1731279
    2     1.583533784     -0.4683497459E-02     401.4645571
    3     1.566091768     -0.4756219852E-02     405.9357838
    4     1.536833771     -0.4880738949E-02     413.6639246
```

5	1.495406212	-0.5064325626E-02	425.1237452
6	1.441236925	-0.5321345817E-02	441.1021382
7	1.373601753	-0.5679268397E-02	462.8216933
8	1.292110096	-0.6199981048E-02	492.0112390
9	1.199679112	-0.7102009626E-02	529.9189451
10	1.118091319	-0.8567449287E-02	568.5874479
11	1.048124773	-0.1438897795E-01	606.5429480
12	0.9134704442	-0.2813311038E-01	695.9532116
13	0.7437697352	-0.5255386447E-01	854.7439608
14	0.5508080071	-0.1253184401	1154.182004
15	0.4228208884	-0.1015384123E-04	1503.550810
16	0.4197996584	-0.3674374630E-04	1514.371621
17	0.4147040877	-0.7083305672E-04	1532.979077
18	0.4074513892	-0.1050619618E-03	1560.266344
19	0.3979549709	-0.1442993821E-03	1597.499054
20	0.3862687552	-0.2587296477E-03	1645.829958
21	0.3746489608	-0.1447114661E-02	1696.875625
22	0.3680576969	-0.1238384841E-02	1727.263673
23	0.3531865357	-0.1175294990E-02	1799.991294
24	0.3395074916	-0.1089632593E-01	1872.514466
25	0.3279617865	-0.5218494342E-02	1938.435255
26	0.3006111365	-0.4452675793E-02	2114.800858
27	0.2918052470	-0.9868425109E-01	2178.619802
28	0.2670993726	-0.6262395959E-02	2380.135465
29	0.2266553113	-0.5840949052E-01	2804.843557
30	0.2219558853	-0.1036669405E-01	2864.229928

5.3 Love problem (coase grid)

```
KRAKENC-Love problem ( coarse grid )

Frequency =    100.0     NMEDIA =    2


    C-LINEAR approximation to SSP
    Attenuation units: dB/mkHz
    VACUUM


      Z          ALPHAR      BETAR       RHO        ALPHAI      BETAI


         ( Number of pts =    500  RMS roughness =  0.000E+00 )
      0.00        1500.00      0.00       1.00        0.0000      0.0000
    100.00        1500.00      0.00       1.00        0.0000      0.0000

         ( Number of pts =    125  RMS roughness =  0.000E+00 )
    100.00        1650.00    400.00       1.90        0.2000      0.4000
    125.00        1650.00    400.00       1.90        0.2000      0.4000

                                     ( RMS roughness =  0.000E+00 )
    ACOUSTO-ELASTIC half-space
    125.00        1800.00    600.00       2.00        0.5000      1.0000

CLOW =    350.00      CHIGH =    4000.0
RMAX =     1000.00000000000

Number of sources   =           1
  70.00000

Number of receivers =           1
  70.00000

Mesh multiplier   CPU seconds
       1            0.841E-01
       2            0.136
       4            0.733E-01
       8            0.101
      16            0.157
```

```
*** WARNING ***

Error detected in program or subroutine: KRAKENC
Too many meshes needed: check convergence
```

I	K	ALPHA	PHASE SPEED
1	1.564905257	-0.4629535024E-02	401.5057958
2	1.547223356	-0.4703537706E-02	406.0942644
3	1.517560371	-0.4830331953E-02	414.0319838
4	1.475550837	-0.5017560289E-02	425.8196431
5	1.420604378	-0.5280418077E-02	442.2895921
6	1.351988258	-0.5648315959E-02	464.7366772
7	1.269405910	-0.6189706598E-02	494.9705416
8	1.176653840	-0.7157937953E-02	533.9875750
9	1.098223525	-0.8541262864E-02	572.1226291
10	1.023013778	-0.1693926004E-01	614.1838402
11	0.4096649356	-0.6991800184E-04	1533.737638
12	0.4023335700	-0.1044559052E-03	1561.685570
13	0.3927340499	-0.1465118290E-03	1599.857539
14	0.3809427647	-0.2765826165E-03	1649.377778
15	0.3696046000	-0.1518107506E-02	1699.974867
16	0.3627090068	-0.1102581777E-02	1732.293709
17	0.3474884731	-0.1397308818E-02	1808.170858
18	0.3342916924	-0.1131406738E-01	1879.551736
19	0.3214561249	-0.5133155098E-02	1954.601210
20	0.2935932884	-0.4512141946E-02	2140.098413
21	0.2592246533	-0.6417767155E-02	2423.837867

```
KRAKENZ-Love problem ( coarse grid )

Frequency =  100.00      NMEDIA =   2


    C-LINEAR approximation to SSP
    Attenuation units: dB/mkHz
    VACUUM


     Z           ALPHAR      BETAR       RHO        ALPHAI     BETAI


          ( Number of pts =    500  RMS roughness =  0.000E+00 )
     0.00        1500.00      0.00       1.00       0.0000     0.0000
   100.00        1500.00      0.00       1.00       0.0000     0.0000

          ( Number of pts =    125  RMS roughness =  0.000E+00 )
   100.00        1650.00    400.00       1.90       0.2000     0.4000
   125.00        1650.00    400.00       1.90       0.2000     0.4000

                                      ( RMS roughness =  0.000E+00 )
    ACOUSTO-ELASTIC half-space
   125.00        1800.00    600.00       2.00       0.5000     1.0000

CLOW =    350.00      CHIGH =    4000.0
RMIN, RMAX =    0.500000000000000        1000.00000000000

Number of sources   =            1
  70.00000

Number of receivers =            1
  70.00000

Mesh multiplier   CPU seconds
       1            0.702
       2            0.164
ISET, MODE =            3           18



*** WARNING ***

Error detected in program or subroutine: KRAKENC-ZSECCX
```

```
 *** FAILURE TO CONVERGE IN SECANT



ISET, MODE =            3          19


*** WARNING ***

Error detected in program or subroutine: KRAKENC-ZSECCX
 *** FAILURE TO CONVERGE IN SECANT



       4              9.09
       8             0.684
      16              1.55


*** WARNING ***

Error detected in program or subroutine: KRAKENC
Too many meshes needed: check convergence



   I         K               ALPHA            PHASE SPEED
   1     1.769048641     -0.5254540902E-02    355.1731231
   2     1.564918878     -0.4629575835E-02    401.5023012
   3     1.547237345     -0.4703582270E-02    406.0905929
   4     1.517575006     -0.4830383692E-02    414.0279908
   5     1.135390419     -0.7637949354E-02    553.3942513
   6     1.064392634     -0.9201874559E-02    590.3071015
   7    0.9900422428     -0.2111940358E-01    634.6381029
   8    0.8173781427     -0.3676704630E-01    768.6999418
   9    0.7101347121     -0.5852240698E-01    884.7878016
  10    0.4178677301     -0.1000691717E-04    1503.630181
  11    0.4148148901     -0.3621319730E-04    1514.696183
  12    0.4096651754     -0.6993180773E-04    1533.736740
```

13	0.3627161597	-0.1101285034E-02	1732.259548
14	0.3347206804	-0.1104724448E-01	1877.142846
15	0.3341500086	-0.2749376573E-02	1880.348689
16	0.2949459890	-0.4758464219E-02	2130.283354
17	0.2309839198	-0.1100866583E-01	2720.182995
18	0.2191096952	-0.5953572240E-01	2867.598032
19	0.2139502914	-0.1068029419E-01	2936.750059
20	0.1466049464	-0.1361849531E-01	4285.793530
21	0.3959932335E-01	-0.3175581855	15866.90069
22	0.3626494682E-01	0.3320377229	17325.78111
23	0.2822895745E-01	-0.6503236576E-01	22257.94317
24	0.1464871165E-01	-0.3134930992	42892.40895
25	0.1096857616E-01	-0.2515783619	57283.50897
26	0.8628035189E-02	-0.4684465934	72822.89849
27	0.7897721228E-02	-0.4161768944	79556.93960
28	0.5812038210E-02	0.3372489649	108106.4005
29	0.3016809170E-02	-0.4038896741	208272.5474

5.4 Fluid sediment (critical frequency)

```
KRAKENC-Fluid sediment problem ( Critical frequency )

Frequency =   88.47     NMEDIA =   2


    C-LINEAR approximation to SSP
    Attenuation units: dB/mkHz
    VACUUM


     Z           ALPHAR      BETAR       RHO         ALPHAI      BETAI


          ( Number of pts =    250  RMS roughness =  0.000E+00 )
     0.00        1500.00      0.00       1.00        0.0000      0.0000
   100.00        1500.00      0.00       1.00        0.0000      0.0000

          ( Number of pts =     64  RMS roughness =  0.000E+00 )
   100.00        1650.00      0.00       1.90        0.2000      0.0000
   125.00        1650.00      0.00       1.90        0.2000      0.0000

                                   ( RMS roughness =  0.000E+00 )
    ACOUSTO-ELASTIC half-space
   125.00        1800.00    600.00       2.00        0.5000      1.0000

 CLOW =    350.00       CHIGH =    4000.0
 RMAX =     1000.00000000000

 Number of sources   =            1
   70.00000

 Number of receivers =            1
   70.00000

 Mesh multiplier   CPU seconds
        1            0.167E-01
        2            0.200E-01
        4            0.500E-02

   I          K                ALPHA           PHASE SPEED
   1   0.3695194718      -0.6317407543E-05    1504.314242
   2   0.3662603630      -0.2229725437E-04    1517.700140
```

3	0.3606388958	-0.4391222769E-04	1541.357326
4	0.3524703375	-0.7381418976E-04	1577.078537
5	0.3416280305	-0.1380641703E-03	1627.130547
6	0.3286113781	-0.5564753281E-03	1691.582949
7	0.3206243543	-0.2021930689E-02	1733.721711
8	0.3088239605	-0.9829474313E-03	1799.968510
9	0.2865960204	-0.3759729195E-02	1939.571259
10	0.2568162869	-0.4964246539E-02	2164.478783
11	0.2196734142	-0.6230517082E-02	2530.453702
12	0.1702475144	-0.9823646161E-02	3265.089690

```
KRAKENZ-Fluid sediment problem ( Critical frequency )

Frequency =  88.470     NMEDIA =    2


    C-LINEAR approximation to SSP
    Attenuation units: dB/mkHz
    VACUUM


     Z           ALPHAR      BETAR       RHO        ALPHAI      BETAI


         ( Number of pts =    250  RMS roughness =  0.000E+00 )
     0.00        1500.00      0.00       1.00       0.0000      0.0000
   100.00        1500.00      0.00       1.00       0.0000      0.0000

         ( Number of pts =     64  RMS roughness =  0.000E+00 )
   100.00        1650.00      0.00       1.90       0.2000      0.0000
   125.00        1650.00      0.00       1.90       0.2000      0.0000

                                 ( RMS roughness =  0.000E+00 )
    ACOUSTO-ELASTIC half-space
   125.00        1800.00    600.00       2.00       0.5000      1.0000

CLOW =    350.00       CHIGH =    4000.0
RMIN, RMAX =     10.0000000000000         1000.00000000000

Number of sources    =               1
  70.00000

Number of receivers =                1
  70.00000

Mesh multiplier    CPU seconds
       1              0.208
       2              0.500E-02
       4              0.750E-02

    I          K                 ALPHA              PHASE SPEED
    1     1.127130253      -0.1261210390E-01      493.1758353
    2    0.5630563240      -0.2281992826          987.2429815
    3    0.3695195097      -0.6320702782E-05      1504.314088
    4    0.3662604970      -0.2230825636E-04      1517.699585
```

5	0.3606391593	-0.4393347226E-04	1541.356200
6	0.3524707790	-0.7385431258E-04	1577.076561
7	0.3416288509	-0.1381834008E-03	1627.126640
8	0.3286147186	-0.5579876502E-03	1691.565754
9	0.3206420010	-0.2022874058E-02	1733.626295
10	0.3088313985	-0.9894995973E-03	1799.925159
11	0.2950457815	-0.2218743336E-01	1884.024240
12	0.2865895713	-0.3755734919E-02	1939.614905
13	0.2602905191	-0.5351101916E-01	2135.588365
14	0.2568205558	-0.4964297376E-02	2164.442805
15	0.2196729685	-0.6234158785E-02	2530.458836
16	0.1702436012	-0.9824163122E-02	3265.164742

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